liarly good omen for progress in the study of nebulæ that a mode of record at once so fluctuating and so laborious as that of hand-drawing should be replaced (as it will no doubt soon wholly be) by automatic impressions which, with some points of inferiority, at least leave no room for "individualism."

M. Tempel's description of the Merope nebula (discovered by himself in 1859), and his remarks on the great elliptical mass in the girdle of Andromeda, derive particular interest from recent events. His observational faculty, and the high quality of his telescope, are illustrated in the disclosure to him, by Amici I., of nearly 900 stars in the Pleiades, all of them comprised within the field of view of a 4-inch Steinheil bearing a magnifying-power of 24. With the former instrument he detected independently on September 30, 1875, a few days before reading Bond's description of them, the strange obscure channels in the Andromeda nebula; and has since with some difficulty made out similar markings in some small nebulæ of the same class. They would accordingly appear to be a more or less characteristic feature of "oval" nebulæ, and might perhaps be assimilated to the symptoms of partial duplication in Messier I.

In respect to the nature of nebulæ, our author's experience leads him decidedly to adopt the view of their close connection with stars. He shows, indeed, for spectroscopic evidence a disregard that is neither philosophic nor just; yet his contention that *purely gaseous* nebulæ do not exist, is probably well founded. No aggregation of celestial mist, at any rate, has ever been observed by him in which his 11-inch failed to reveal the pricking light of minute stars, marking some knot or nucleus, and thereby evincing structural relations of a most intimate kind.

THE MATHEMATICAL TRIPOS[1]

III.

When the interval between the earlier portion of the examination and Part III. had been extended to a year, it became evident that some substantial relief must be afforded to the examiners. By the existing regulations a person who accepted the office of Moderator would have to take part in the examination in three consecutive years, and in his second year of office he would have to examine the candidates of one year in Part III. simultaneously with those of the year below in Parts I. and II. This led to the consideration of the whole question of the appointment of examiners. The two Moderators in each year are nominated by two colleges, according to a prescribed cycle of fifty years. This nomination by colleges, though theoretically not very defensible, had worked very fairly so long as the examination only included subjects with which any high wrangler might be expected to be acquainted; but it was clearly unsuitable for Part III. In any case the nomination of the four examiners by four independent bodies might easily bring about the result that among the various subjects included there would be some which had not been made the object of special study by any of the examiners: indeed there was nothing to prevent the four examiners being all pure mathematicians or all physicists. Accordingly, with a unanimity almost unique in matters relating to the Tripos, the Board recommended in a Report dated June 15, 1885, that the examiners for Part III. should be quite distinct from those for Parts I. and II., and that all four should be nominated by the Board. It was also proposed that they should hold office for only one year. This Report was sanctioned by the Senate on October 29, 1886. In future, therefore, the Moderators will not take part in the highest portion of the examination. The appointment of Moderators dates from 1680. Previously the Proctors had themselves presided in the schools, but in that year the duty of conducting the disputations was transferred to the Moderators, who were specially appointed to perform this office. The Moderators have always been, and still remain, high University officers, ranking next to the Proctors.[1] Not only were they the earliest examiners in the University, but it is to them that we owe the origin of the examination system. Their severance from a portion—and that the highest portion—of the examination is therefore a notable event in the history of the Tripos. Neither the Board nor the University would have agreed lightly to such a break in the traditions of the Senate House examination, had it been possible to retain the Moderators as examiners for the final part without altering the system of nomination by colleges. The complete separation of Part III. from the earlier parts of the examination was, however, inevitable. Many members of the University who would discharge most admirably the duty of examining for Parts I. and II. would shrink from Part III.; and the professors and specialists who were best fitted to examine in Part III. would generally be reluctant to undertake the heavy burden of examining all the candidates for Parts I. and II., especially in two consecutive years.

Thus, by the irresistible pressure of events, it has come to pass, in the last few years, that not only the titles of wranglers, senior optimes, and junior optimes have lost their old significance and refer only to the earlier examination, but that even the more ancient title and office of Moderator has undergone a similar restriction. The final part of the examination has indeed made rapid progress: within three years of its first coming into existence it has emancipated itself from union with the earlier parts, and become an independent examination.

Besides these important innovations, the Senate sanctioned at the same time a slight change in the nomenclature of the Tripos, the earlier portion of the examination, previously called Parts I. and II., upon which the list in order of merit depended, being designated Part I., and Part III. being henceforth designated Part II. This change was made in order to bring the nomenclature of the Mathematical Tripos into harmony with that of the Classical and other divided Triposes.

As soon as Part II., to adopt its new name, became an independent examination, the Board directed its attention to the schedule of subjects relating to it. The existing schedule contained only those subjects which had been included in the schedule which came into operation in 1873, when the results of the whole examination were still expressed by one final list, arranged in order of merit. Now that Part II. was a separate examination, and that there was no order of merit, the reasons for the limitation of the subjects had been entirely removed. Although the theory of elliptic functions, which dates only from the publication of Jacobi's "Fundamenta Nova" in 1829, was included, the theory of numbers, which had its origin in Gauss's "Disquisitiones Arithmeticæ" of 1801, was still excluded. Abelian functions, the theory of functions of a complex variable, projective geometry, and quaternions were not formally included by name, and questions on these subjects could only be set, if at all, by straining the meaning of the title of some other subject. Besides the total exclusion of certain branches of pure mathematics, a further reason for revising the existing schedule was afforded by the fact that the four groups A, B, C, D were very unequal both in magnitude and popularity among the students. According to the existing regulations the four groups had to be equally represented by questions, and this led to a great waste of examining power, many questions having to be constructed each year upon subjects which none of the candidates had studied. The Board accordingly formed a schedule in which all the subjects of pure and applied mathematics were included—none being intentionally omitted. These were divided into eight divisions, the first four relating to pure mathematics, and the last four to applied mathematics. To avoid the waste of questions that would ensue from the examiners having to represent all the subjects in the papers in each year, they proposed that before the first day of December preceding the examination the names of the candidates and of the divisions and subjects in which they desired to be examined should be forwarded to the Registrary of the University. The examiners would thus be made acquainted with the subjects which the candidates had studied, and would be able to frame their questions accordingly. Changes were also proposed with respect to the candidates who were admissible, and to the form of the final list. By the existing regulations only wranglers were allowed to present themselves for examination in Part II., and the list was arranged in three divisions; there was no separation into *classes*, because, as only wranglers were admissible, it was considered that all the candidates were first-class men from the beginning. The new proposals were, that the restriction which admits only wranglers should be removed, and that the candidates should be divided into three classes, each class being subdivided into as many divisions as the examiners in each year thought proper. In previous schemes the endeavour of the Board had been to frame regulations that would tempt the students to specialise their reading. A few years had made so great a difference that, with a view to prevent undue specialisation, the Board now inserted a regulation to the effect that proficiency in subjects taken from more than one of the divisions should be requisite in order that a candidate might be placed in the first class.

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[1] Address delivered before the London Mathematical Society by the President, Mr. J. W. L. Glaisher, M.A., F.R.S., on vacating the chair, November 11, 1886. Continued from p. 157.

[1] It is still the custom for the list of wranglers, senior optimes, and junior optimes to be shown to the senior Proctor on the evening of the day before it is read in the Senate House. This is doubtless a relic of the fact that the Moderators were originally the substitutes of the Proctors. It has been already mentioned that the Proctors, as well as the Vice-Chancellor, used to have the right to insert a certain number of names where they pleased in the Tripos list.

The Report containing these proposals was confirmed by the Senate on May 27, and it is noteworthy that both this Report and its predecessor, in which the nomination of examiners was placed in the hands of the Board, were sanctioned without opposition of any kind. The latter of these Reports also made a few minor changes, the most important of which was the omission of the problem paper which had been still retained, from the old five days, in the scheme of 1882.[1] The examination in Part II. had assumed such a character, that the kind of questions to which one would usually apply the name of problems was no longer in keeping with the contents of the other papers.[2]

Under the new scheme, in which all the examiners were to be appointed by the Board with special reference to their collective fitness for conducting the examination in Part II., there was no further need for an Additional Examiner, and this office was, accordingly, discontinued.[3]

Thus has the Mathematical Tripos been divided into two parts; and thus has surely arisen in the University a mathematical examination of a higher type than has been known before, or could have existed under any system in which all the candidates for mathematical honours were required to be examined by the same papers throughout. For those who study mathematics for the sake of exact knowledge or mental discipline, and who propose to go forth into the world to follow professional or other careers, the first part secures all the old stimulus to industry, and gives to those who are successful the same stamp of intellectual distinction as before: such students are released at the end of their third year to enter upon the active duties of their lives, equipped with a sound understanding of the principles of the exact sciences, and with minds well trained to accurate habits in reasoning and in the acquisition of knowledge. To those whose attachment to our science lies deeper, and whose studies have carried them beyond its threshold, the second part, at the end of their fourth year, affords an opportunity of distinction of a higher kind, and one more suited to their tastes: no longer is the wise and thoughtful student hopelessly distanced in the Tripos race by his quick and ready rival.

The wants of the candidate whose mathematical career closes with the last paper in Part I., and of the candidate whose mathematical life only begins from this moment, are equally provided for by the new scheme. The order of merit relates to an examination that can bear it. All the subjects included in Part I. are such as ought to be the common property of every one who has received a sound mathematical education; and by the results of an examination in subjects which all the candidates should have read a list in order of merit can properly be formed. The specialist for the first time is set free to follow his own tastes, and give his whole heart and time to the branches of mathematics by which he is attracted. The University permits him to select any subjects he pleases from the whole range of pure and applied mathematics, and undertakes to examine him in them and award to him the credit he deserves for his attainments. A perfectly free choice is given to him, subject only to the one condition that, in order to qualify himself for admission to the first class, he must not select all his subjects from a single division.[1]

But what to us as mathematicians is more than all, as bearing on the future of our science, is that now for the first time will it be possible in Cambridge for an able and earnest worker and teacher to interest and engage his pupils in his work, and found a school such as we are so familiar with in foreign Universities, where the presence of a great professor has been almost invariably marked by a succession of illustrious pupils—pupils worthy of their master, and worthy to carry on his work. Think of the school of arithmeticians founded by Gauss at Göttingen, and how impossible such a result would have been at Cambridge, dominated as she has been by the competition for places in the Tripos! Great as has been the value to the University of the order of merit—as a stimulus to industry, an encouragement to thoroughness in mathematical study, and a paramount influence in regulating elections to Fellowships at colleges where no independent examination existed—it has yet been in recent years a deadly enemy to the spread of research and the advance of our science. Throughout his whole career the student has had to devote himself unremittingly to the work for his Tripos, taking up a fresh subject each term, and often having to read two in one term. He could never pursue any subject far enough to reach the really interesting portions of it, or obtain complete command over its methods: he was always occupied with something new, starting afresh and gaining familiarity with new principles, new processes, new modes of thought. Many of the higher lecturers in the University were necessarily neglected by the students: they could pay but scant attention to any subject which was not adequately represented in the Tripos, and even in the case of the subjects which were so represented they were tempted to pass lightly over those investigations, however important, which from their length and character were unsuitable for reproduction in an examination. Now, however, all this is history. When a good course of lectures upon any high subject is given in the University, those students who have attended the course will send in that subject as one on which they desire to be examined: it will, therefore, be properly represented by questions; and the subject will become one that will be increasingly studied year by year. It will now be possible for any capable mathematician, by means of his lectures, to gather pupils round him who will bring his subject into prominence, and make it one of special study in the University.[1] It has been said that in mathematics we have in England generals without armies: the great men who are independent of circumstances have arisen among us, but where are the rank and file? It is my belief that the great obstacle to the existence of the rank and file has now been removed.

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[1] Although the Board were unable to make any recommendation upon the subject, I may mention that the principle of prefixing to the final three days a preliminary day, in which the subjects of examination should be those parts of higher pure mathematics which are needed in mathematical physics, found a considerable amount of favour on the Board. The proposal, however, was found to be more difficult in execution than was anticipated (partly on account of the impossibility of forming a perfectly satisfactory schedule of subjects for this day), and was ultimately abandoned by most of its original supporters.

[2] In the first examination in Part III., in 1883, the examiners set, as one of the question papers, a paper of essays; and their example was followed by the examiners in 1884, 1885, and 1886. These essay papers were introduced merely on the authority of the examiners, and not in consequence of any new regulation. Experience seems to show that the essay paper affords very little additional assistance in ascertaining the relative merits of the candidates. The essays were, perhaps, more useful at first, when they were a novelty.

[3] Unless the office should be revived at some future time, there will therefore have been only one Additional Examiner for the final part of the examination, viz. in 1886, the last occasion of the examination taking place in January.

[1] This condition would be complied with by the candidate's showing proficiency in one subject taken from one division and in one other subject taken from one other division. The intention of the Board was to discourage students from specialising too narrowly at too early a period. Some of the divisions (as, for example, the fourth, which contains only projective geometry and analytical geometry of curves and surfaces) are so restricted that it was considered undesirable that students should be allowed to confine themselves entirely to subjects chosen from a single division.

Whatever else it may be, Part II. is at all events a "limiting form." No wider choice of subjects could be given to the candidates; no greater freedom to the examiners. The schedule of subjects includes all mathematics: the examiners may issue any kind of list. By introducing numerous divisions into the classes they may make it approximate as closely as they please to an order of merit; or, on the other hand, they may make it merely a class list. They are empowered to give to their list just such a form as they feel justified in doing by the results of the examination. In the appointment of examiners, also, the limiting form has been reached, all four being nominated by the same University authority, and holding office for one year only.

With respect to Part I., it may be that the ultimate form has not yet been reached. There are some who think that, as in some other Triposes, the students should have the option of becoming candidates at the end of their second year. It would seem, also, that the range of subjects is rather too restricted; and, as may be inferred from what I have said near the beginning of my address, I should myself like to see the elementary portions of elliptic functions included in the schedule of Part I. Still, these are but minor points; and I think that the principle of subjecting all the candidates for mathematical honours to one and the same examination in comparatively elementary subjects, and arranging the list in order of merit, meets with general approval.

A few years ago, when the old Tripos was exerting its stifling influence upon the higher mathematical studies of the Universities, I felt disposed to welcome the abolition of the order of merit as the lesser of two evils; but now that the Tripos is divided, and that the mathematician has his own examination especially framed for him, I should be sorry to see a modified class list substituted for the order of merit in Part I. A severe competition for places has the great advantages of keeping the candidates closely employed, and extracting from them their best work. At present an immense amount of thoroughly good mathematical work is done in the University. We have received from our predecessors a system under which the principles of mathematics are efficiently taught, the powers of the students are exerted to the utmost, and upwards of a hundred persons each year receive a mathematical education which is in some respects unique. These are substantial advantages which should not lightly be jeopardised or exchanged for others that are problematical. Under any other system I think the quantity and the quality of the mathematical work done in the University would suffer. It should be remembered also that there is no subject in which the knowledge of a candidate can be so readily tested by examination as in mathematics, and that in no other subject can the results of an examination be expressed with such certainty and accuracy by an order of merit.

I believe there are indeed but very few who have graduated in the Tripos who would set a slight value upon the advantage which their mathematical training has been to them throughout life; and on the other hand I think that it has been an indirect benefit to our science that among those who have won distinction in public and professional life there have always been some—and those not the least influential or eminent—who have passed through an extensive and thorough course of mathematical study, and to whom our world of symbols is no *terra incognita*. The fact that our results, unlike the conquests of astronomy and other branches of applied mathematics, can only be expressed by means of a language of their own, requiring years of study, imposes of necessity such narrow limitations upon the numbers of our audience that we cannot be insensible to the advantages of any system by which the power of understanding and appreciating the beauties of our science is extended to others external to our own ranks. Under the new scheme these advantages are still retained; and, difficult as is the problem of combining a mathematical course for the many with the technical requirements of the few, I believe that a satisfactory solution has rewarded the efforts of the last twenty years. I believe that the University of Cambridge will become a great centre of mathematical research and a home of the exact sciences, and that it will be found that these objects have been attained without any sacrifice of the general efficiency of the training received by the bulk of the candidates for mathematical honours.

On taking a survey of the history of the Tripos during the last half century, perhaps the feature that stands out most strongly is the part played by the subjects of electricity and magnetism—their half-recognised existence before 1848, their exclusion until 1873, and the effects which followed their restoration in that year. It was the extension of the dominion of mathematics over these great and growing branches of physical science that broke down the old system. Electricity and magnetism became too important to be excluded; but when included the examination in its old form was too heavily weighted to exist.

The year 1877-78, in which the syndicate of 1877 was endeavouring to frame a scheme that should relieve the strain of the excessive competition without sacrificing the order of merit, was perhaps the most eventful period in the whole history of the examination: it then became evident that it was impossible to retain the existing system even in a modified form, and that a complete reorganisation of some kind was inevitable. Although the frequent changes in the last few years have been productive of some inconvenience, I think it is fortunate that the syndicate was so reluctant to propose any sweeping changes, and that the present scheme has come into existence as it has done—not as the work of any influential legislator, but as the form which the examination has of itself assumed under the pressure of the actual forces at work in the University. The order of merit for the whole examination was not given up till it was clearly shown that its retention was an impossibility; and, on the other hand, Part II. has grown up by a process of regular development, and been moulded into its present form by those most interested in promoting the higher mathematical studies of the University.

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[1] In the schedule for Part II. no subjects are ignored or favoured less than others, so that by the new scheme provision is made for the growth of any subject which may happen to take root.

Special reference also should be made to the "three days." It will be remembered that this preliminary portion of the examination was the principal feature of the scheme which came into operation in 1848. Both the subjects and the methods of solution that may be employed are defined by a schedule, and only those who satisfy the examiners by their performances in these three days are admitted to the subsequent parts of the examination. It is very singular that an arrangement devised so long ago should not only still continue in force, but even be regarded by some as the most thoroughly satisfactory portion of the whole system. The framers of this scheme and schedule might well have been proud of the lasting character of their work if they could have known that it would outlive two sets of University statutes, and, amidst changes on every side, remain unchanged for forty years.

The early history of the Tripos and its gradual development into an examination by written answers, and finally by printed papers also, are especially interesting in these days, when the merits of the examination system are so highly appreciated, and its adoption is so universal. The Senate House at Cambridge is the cradle of the modern form of examination in England.

In connection with the Tripos there is one matter of so much importance that I cannot pass over it entirely without mention. I mean the influence of the system of private tuition. I believe that while there is an order of merit it will always be a great assistance to the majority of the candidates to read with a private tutor. Mathematics is a difficult science; and when a considerable range of subjects has to be traversed in a comparatively short time, and the knowledge of the candidate has to be finally tested by an examination such as Part I. of the Tripos, it cannot fail to be a great advantage to him to have his difficulties explained, his path smoothed, and his skill in working out problems developed, by an experienced private tutor specially interested in his individual welfare. The system of private tuition has been objected to from two points of view: (1) because it is unsatisfactory that the instruction which is valued most highly by the student should be received from his private tutor instead of from college lecturers or University professors; and (2) because the student who has followed implicitly, during his whole undergraduate career, the minute directions of one man with regard to his reading, is placed after his degree, when he is deprived of his guide, in a very unfavourable position for pursuing further his mathematical studies. The first objection does not concern us here: my own feeling is, as I have just said, that, whenever an order of merit exists and the competition for places is keen, the services of private tutors will necessarily be called into requisition. The second objection is one which is of far more importance to our science. There can be no question that, brilliant and eminent as have been the greatest of the private tutors at Cambridge, one result of the system has been that many of the ablest students have been left after graduation not only without any knowledge of the way to follow up the study of the subjects of which they had learned the elements, but even without any taste or inclination to do so. The private tutor's manuscript and verbal instruction had superseded all need of referring to the original memoirs, and the nascent wrangler knew nothing of the great world of mathematical literature or of the modes of reaching it. On the other hand, it is only fair to say that the amount of mathematical knowledge acquired by the best pupils from their private tutors in the course of their undergraduate career was really wonderful; and that till quite recently neither the University nor the colleges offered any inducement to the mathematical student to continue his reading after the Tripos. The fact that the student's horizon should have been bounded by the Tripos, and that his training should have been directed with the view to giving him skill in working out questions rather than to developing his taste for the science he was studying, was principally the fault of the system as a whole; but it was certainly intensified by the complete subjection of the pupil to the course of reading placed before him by the private tutor. A student whose interests and aspirations had been at least held in check, and perhaps entirely stifled, throughout his whole undergraduate career, was generally too subdued or helpless to be able to make use of his freedom when the examination was over.

Under the new scheme the private tutor still occupies in the main his old position with respect to Part I., although, of course, the higher places in this examination have much less significance than before. With respect to Part II. it is quite different. No attempt is made by private tutors to teach these higher subjects, which, both from their character and extent, are clearly unsuited for private tuition; and the students are compelled to rely upon the lectures delivered in the colleges and University in their preparation for this final examination. Thus, in their fourth year they are brought into contact with the leading mathematicians in Cambridge; and when the examination sets them free to pursue their own studies or researches, they start on their new career fresh from the best teaching which the University affords.

Although the subject of my address is the Mathematical Tripos, it may be regarded as still falling within my province to refer to other changes that have taken place in the University for the purpose of encouraging original mathematical work. Fourteen years ago Trinity College invited mathematical candidates for Fellowships to send in, before the examination, dissertations upon any subjects of their own selection. It was announced that these dissertations, if possessed of decided merit, would be taken into account in the Fellowship election, together with the results of the Fellowship examination. Not only have these dissertations been of the greatest value in guiding the choice of the electors, but many of them have been important contributions to mathematical literature.[1] The example of Trinity College has been recently followed by St. John's College and King's College. The Smith's Prizes, which for a great number of years had been awarded by a special examination, are now awarded annually for mathematical dissertations, the candidates being free to select their own subjects. This new scheme passed the Senate on October 25, 1883, and the first award of the prizes under it was made in 1885. Powerful inducements are, therefore, now held out by the University and some of the colleges for the best students to devote themselves to original work. The importance to our science of these direct incentives to research cannot be overestimated. They come into operation as soon as the stimulus of the examination is removed, and, instead of resting upon their laurels, the ablest mathematicians of the year are induced to concentrate their powers upon a single subject, just at the time when they are undaunted by any amount of hard work, when their stock of general mathematical knowledge is freshly acquired, and when their minds are flexible, vigorous, and free from care. It is indeed strange to look back upon the changes of the last few years, and to contrast the encouragement now given to mathematical research with the indifference, or even worse, of twenty years ago.[1]

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[1] Among the Trinity dissertations which have subsequently been printed, I may mention the late Mr. R. C. Rowe's "Memoir on Abel's Theorem" (*Phil. Trans.*, 1881), Mr. Forsyth's "Memoir on the Theta Functions" (*Phil. Trans.*, 1882), Mr. Homersham Cox's "Application of Quaternions and Grassmann's Ausdehnungslehre to different kinds of Uniform Space" (*Camb. Trans.*, 1882), Mr. Gallop's "Distribution of Electricity on the Circular Disc and Spherical Bowl" (*Quart. Math. Journ.*, 1886), and Mr. R. Lachlan's "Systems of Circles and Spheres" (*Phil. Trans.*, 1886).

I cannot close my address without saying a few words upon our Society. We were founded in 1865, and so today we attain our majority. I think we can safely say that we have steadily and uniformly kept to our single purpose of promoting the advance of mathematics. We have published seventeen volumes of *Proceedings*, and every paper we have printed has been subjected to a rigorous examination by two referees. We already have a history to look back upon: familiar presences among us—De Morgan, Clerk Maxwell, Clifford, Henry J. S. Smith, Spottiswoode—have passed away; and for most of us this very room is full of associations with those whom we shall see no more. I should like before concluding to formally express our gratitude to our two Secretaries, Mr. Morgan Jenkins and Mr. Robert Tucker, who have served us so faithfully almost from our foundation, and to whom the successful development of our Society has been largely due. I will not utter any aspirations with regard to our future: we shall never be a great Society in numbers, but we can continue to do what we have done, and to spare no effort to encourage the advance of mathematical science.

THE INTERNATIONAL COMMITTEE OF WEIGHTS AND MEASURES

THE results of the scientific investigations made under the directions of the Comité International des Poids et Mesures at their Bureau at Sèvres during the past year are stated in vol. v. of their "Travaux et Mémoires," recently published (Gauthier-Villars, Paris, 1886) under the authority of the Director of the Bureau. This volume contains the following papers:—"Note sur l'étallonage des sous-divisions d'une règle, sur l'étude des erreurs progressives d'une vis micrométrique, et sur le calibrage des thermomètres," by Dr. O. J. Broch; "Études thermométriques," by M. Ch. Ed. Guillaume; "Études sur la balance," by Dr. M. Thiesen; "Sur quelques analyses chimiques faites pour le Bureau International," by M. Tornöe. The two latter memoirs, however, are published under the responsibility of their authors.

In the method of calculating the errors of the subdivisions of a standard measure of length, or of calibrating a thermometer, Dr. Broch has followed the celebrated astronomer P. A. Hansen; but he has endeavoured to render Hansen's method more simple; and he has abbreviated it and reduced the number of observations, without increasing the probable error of the results obtained. Convenient tables of equations are given for the more ready application of Hansen's formula, and also examples of an abbreviated method for calculating the several lengths of the decimetres, centimetres, and millimetres on a subdivided standard metre.

The second part of this Note deals with the progressive errors of the micrometer-screw. As each interval to be measured on a linear standard is contained within two lines, we have to pass by successive turns of the micrometer-screw from one line to the other. Each line in turn is bisected by means of cross or of parallel webs; and not only do the personal errors of bisection have, of course, to be allowed for, but even small errors in the micrometer-screw itself have to be corrected. Examples of such corrections are given in this note. In considering the progressive errors of micrometer-screws, we are not sure that Dr. Broch has sufficiently, however, investigated the variation in the amounts of such errors owing to the wear of the screw.

In the third part of the note is given an explanation of an abbreviated method of calibrating graduated glass tubes or thermometer-stems, and of applying corrections to the calibrated lines. The method of interpolation by differences is also discussed and simplified.

Dr. Guillaume, in his "Études thermométriques," continues the thermometric work which was begun by Dr. Benoit, and by Pernet at this Bureau. It is required of all standard thermometers verified at the Bureau, that they should carry the fundamental points 0° and 100° C.; that they should have a total length of as much as 70 centimetres, the diameter of the stem varying from 3·5 to 5·5 millimetres; and that each division should be nearly 5 millimetres in length. The testing of the thermometers includes the three distinct operations:—

(1) Division and calibration.

(2) Determination of the coefficient of pressure (when the thermometer is placed alternately in a vertical and in a horizontal position).

(3) Determination of the fundamental points and of the mean value of each degree.

It is with these operations, as well as with the actual verification of certain standard thermometers at the Bureau since the year 1883, that Dr. Guillaume now deals. The paper is an interesting one, and all the observations are printed in the fullest detail. Particularly in that part of this paper which discusses the variations in the readings of thermometers by time and circumstance, there is much to be learnt.

Of late, attention has been given, especially in Germany, to the kind of glass best adapted for thermometers, the zero-points of thermometers made of some kinds of glass, being found less likely to alter by age like ordinary thermometers. A careful analysis made by M. Tornöe of the glass used for the bulbs of two of Tonnelot's thermometers used at the Bureau, gave the following results:—

	Hard glass	Plate glass
Silica	71·52	60·68
Sulphuric acid	0·72	0·37
Chlorine	traces	—
Peroxide of iron	0·22	—
Lime	14·55	5·44
Soda	10·81	10·50
Potash	0·37	6·55
Magnesia	traces	traces
Protoxide of manganese	traces	—
Oxide of lead	—	15·12
Alumina	—	—
Alumina, with traces of iron and manganese	—	0·87

The analyses of the stems of the thermometers showed somewhat different results.

The memoir by Dr. Thiesen, "Études sur la balance," continues the excellent work on the construction and use of the balance which was originally begun at the Bureau by M. Marek. In the "Théorie générale de l'équilibre statique de la balance," and in the "Calcul de l'équilibre de la balance," Dr. Thiesen has discussed the conditions which affect the equilibrity of a balance, and also has investigated the effects of outside influences during weighings, as those arising from currents of air and from changes in the condition of the air; and from electrical disturbances, magnetic and radiometric. A good balance may be relied on to 0·001 mgr. in the comparison of two standard kilogramme weights, but outside influences increase the probable error to ± 0·004 mgr.

The labours of the Bureau have been particularly devoted to the perfecting of existing methods, and they have resulted in the attainment of far higher accuracy in weighing and measuring than was thought to be possible, or necessary, even ten years ago. We trust that the labours of the Comité may soon be crowned by the completion of the international metric standards of length and weight, for which all their present investigations are preparatory.

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[1] Under the old system, the Cambridge graduate who devoted himself to mathematical research possessed one advantage over his Continental colleagues in the wider range of his general mathematical knowledge. Although Part I. is considerably more restricted than the Tripos of 1848-72, this advantage is still retained to a substantial extent.